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Volume XVI, Number 1

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The benefits of education and of useful knowledge, generally diffused through a community, are essential to the preservation of a free government.

Sam Houston

Cultivated mind is the guardian genius of Democracy, and while guided and controlled by virtue, the noblest attribute of man. It is the only dictator that freemen acknowledge, and the only security which freemen desire.

Mirabeau B. Lamar

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THE TEXAS MATHEMATICS TEACHERS' BULLETIN

Volume XVI, Number 1

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Teachers of mathematics in Texas are cordially invited to use this bulletin for the expression of their views. The editors assume no responsibility for statements of facts or opinions in the articles.

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IDENTITIES CONNECTING THE ANGLES OF A TRIANGLE

BY S. B. RED

Students of mathematics are familiar with the conventional methods of proving the important identities of trigonometry. Some of these identities are simple and are easy to prove, while others are complicated and the proofs of them often give the students considerable trouble. The purpose of this article is to show how some of the more complicated ones can be proved in a very simple way. In the development of the identities, the reader will note that there is a departure from the conventional methods in only a few cases. In every case of departure, however, proofs which ordinarily involve complex trigonometric substitutions, resulting in expressions very difficult for the student to follow, are replaced by exceedingly easy proofs.

Consider the triangle *ABC*, Fig. 1.

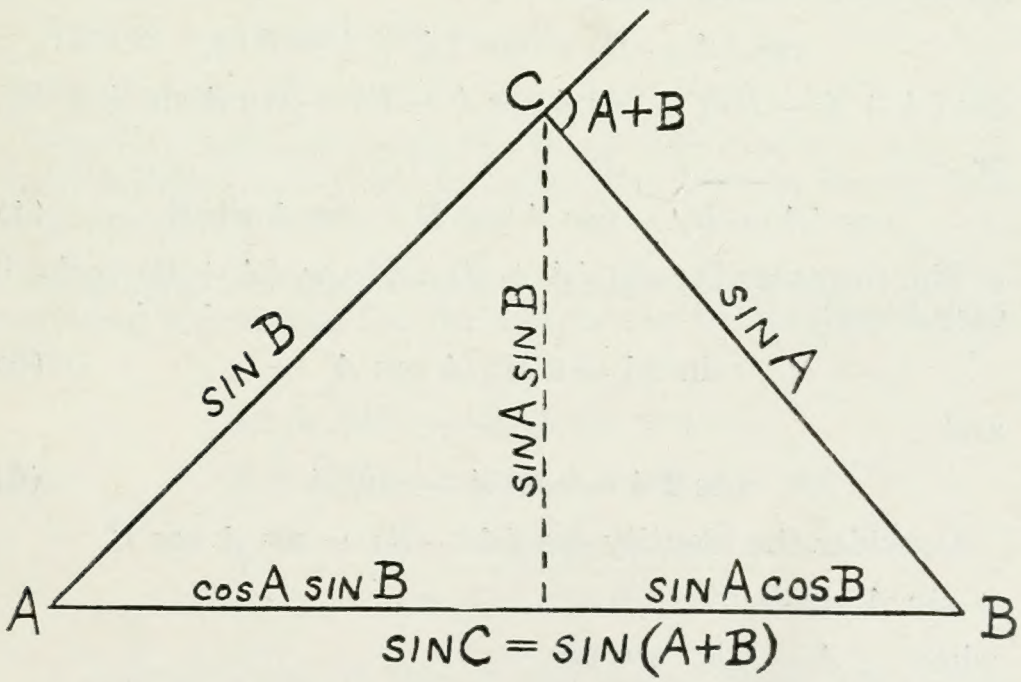

FIG. 1

It follows at once from the figure that $\sin (A + B) = \sin A \cos B + \cos A \sin B.$ (1)

From this identity we can easily obtain the formula for $\sin (A - B)$.

$$\sin (A - B) = \sin [A + (-B)]$$

$$\sin [A + (-B)] = \sin A \cos (-B) + \cos A \sin (-B)$$

or

$$\sin (A - B) = \sin A \cos B - \cos A \sin B. \quad (2)$$

The formula for $\cos (A + B)$ can now be obtained as follows:

$$\cos (A + B) = \sin [90^\circ - (A + B)] = \sin [(90^\circ - A) - B]$$

$$\sin [(90^\circ - A) - B] = \sin (90^\circ - A) \cos B - \cos (90^\circ - A) \sin B.$$

or

$$\cos (A + B) = \cos A \cos B - \sin A \sin B \quad (3)$$

From this identity we may obtain the formula for $\cos (A - B)$ as shown below:

$$\cos (A - B) = \cos [A + (-B)].$$

$$\cos [A + (-B)] = \cos A \cos (-B) - \sin A \sin (-B).$$

or

$$\cos (A - B) = \cos A \cos B + \sin A \sin B \quad (4)$$

The formulas for $\sin (A + B)$ and $\cos (A + B)$ make it easy to get

$$\sin 2A = 2 \sin A \cos A \quad (5)$$

and

$$\cos 2A = \cos^2 A - \sin^2 A \quad (6)$$

Consider the identity $\cos (A - B) = \cos A \cos B + \sin A \sin B$;

when $\angle A = \angle B$ we get

$$\cos (A - A) = \cos^2 A + \sin^2 A$$

But

$$\cos (A - A) = \cos 0 = 1$$

Hence $$1 = \cos^2 A + \sin^2 A \qquad (7)$$

From (7) we can easily get

$$\sec^2 A = 1 + \tan^2 A \qquad (8)$$

by dividing by $\cos^2 A$, and

$$\csc^2 A = 1 + \cot^2 A \qquad (9)$$

by dividing by $\sin^2 A$.

By applying the law of cosines in Fig. 1 we get

$$\sin^2 A = \sin^2 B + \sin^2 C - 2 \sin C \cos A \sin B \qquad (10)$$

$$\sin^2 B = \sin^2 A + \sin^2 C - 2 \sin C \sin A \cos B \qquad (11)$$

$$\sin^2 C = \sin^2 A + \sin^2 B - 2 \sin B \sin A \cos C \qquad (12)$$

By adding (10), (11), and (12) and collecting terms we get

$$\sin^2 A + \sin^2 B + \sin^2 C = 2 \sin C (\cos A \sin B + \sin A \cos B) + 2 \sin B \sin A \cos C$$

$$= 2 \sin^2 C + 2 \sin B \sin A \cos C$$

or

$$\sin^2 A + \sin^2 B - \sin^2 C = 2 \sin B \sin A \cos C.$$

Adding 1 to both sides we get

$$\sin^2 A + \sin^2 B + \cos^2 C = 1 + 2 \sin B \sin A \cos C \qquad (13)$$

The conventional proofs for these identities are very difficult for the student, but by using Fig. 1 it can be seen that they are very simple.

If a circle is inscribed in the triangle ABC, Fig. 1, an interesting expression for the radius can be obtained as follows:

$$2 \triangle ABC = \sin A \sin B \sin C$$

Also $$2 \triangle ABC = r (\sin A + \sin B + \sin C)$$

$$\therefore \quad r = \frac{\sin A \sin B \sin C}{\sin A + \sin B + \sin C}$$

A careful study of Fig. 2 will reveal other interesting identities.

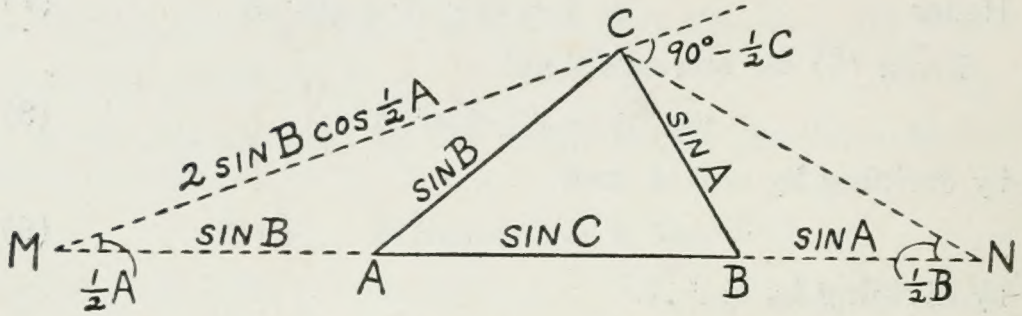

FIG. 2

Consider the triangle MNC. By the law of sines we get

$$\frac{\sin A + \sin B + \sin C}{\cos \frac{1}{2} C} = \frac{2 \sin B \cos \frac{1}{2} A}{\sin \frac{1}{2} B}$$

$$= \frac{4 \sin \frac{1}{2} B \cos \frac{1}{2} B \cos \frac{1}{2} A}{\sin \frac{1}{2} B}$$

$$\therefore \quad \sin A + \sin B + \sin C = 4 \cos \frac{1}{2} A \cos \frac{1}{2} B \cos \frac{1}{2} C$$

If we lay off $AM' = MA$ on AB and join M' to C, the law of sines applied to the triangle $M'NC$ yields similarly the identity

$$\sin A + \sin C - \sin B = 4 \sin \frac{1}{2} A \sin \frac{1}{2} C \cos \frac{1}{2} B$$

Thus we see that the law of sines employed with Fig. 2 yields, in a simple way, some of the most complicated identities of trigonometry.

SOME RECENT TRENDS AND APPLICATIONS OF STATISTICS[1]

BY EDWARD L. DODD
The University of Texas

Statistics! What is drier than a lot of statistics? When we pick up a magazine and find page after page of an article covered with figures and figures and more figures, we drop it to read something else. Perhaps the author needed to develop his subject just as he did with all numerical results interspersed, or perhaps he could have consolidated the data into an appendix. At all events, this brings us to the first point that I wish to stress this afternoon.

In the modern conception, the art of statistics is an art of *condensation.* It is not the detailed elaboration of all information, relevant or irrelevant, that may be on hand. Rather—it is the art of sifting out the irrelevant, the art of *characterizing* by a few important numbers, a vast horde of numbers too numerous for any human mind to hold.

I wonder how many here can tell me what a *statistic* is. You have heard of statistics in the plural. Have you heard of an individual *statistic?* Today, we speak of the average duration of human life as a statistic. Another statistic is the coefficient of correlation of ability in mathematics as connected with ability in physics for specified groups of individuals. A statistic is a number—in general computed from a fairly large aggregate of numbers—which in some measure characterizes the whole aggregate. One problem in modern theory is to find out *what statistics give us the most information in the most condensed form.*

Before delving into theory, however, I want to mention some comparatively new forms of statistical enquiry. For some two hundred years we have had mortality or life tables—tables for the English, tables for Germans, French, Americans, etc. The American Experience Table, legalized

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[1] An address before the Texas Section of The Mathematical Association of America, meeting in Austin, January 30, 1932.

by Texas and most states of the United States, starts as follows:

Age x	Surviving at age x	Dying at age x
10	100,000	749
11	99,251	746

This indicates that of 100,000 individuals alive at age 10, there die on the average, during the year, about 749, leaving 99,251 alive at age 11. This table was based upon the actual experience of the Mutual Life Insurance Company of New York. But, in the early days, various theories were advanced, now known to be inadequate, with regard to the course of a mortality table. One theory was that the number of annual deaths from an original group would be constant; by another theory a constant percentage would die annually. Those familiar with the theory of depreciation of physical assets may see here an analogy with the straight line and the constant percentage methods of estimating depreciation. But some years ago it became clear that for accurate accounting of *depreciation* values, mortality tables should be constructed for *physical units.* I have not, myself, seen extensive *printed* mortality tables for such units. But a *mimeographed* copy of a mortality table for the machine parts of cream separators was sent to me about the year 1920. The theory developed in this connection was somewhat complicated. It corresponded to the theory of increasing populations, whereas the usual mortality table — such as the American Experience — is applicable directly to the case of stationary populations, with annual deaths equaling births. A number of mathematicians and statisticians, including James W. Glover and Arne Fisher, took part in developing the theory of mortality tables for inanimate objects. An important impetus to this work was the desire of the New York Telephone Company to obtain more accurate methods of estimating depreciation, in connection with assessments for taxation. Dr. Glover reviews in the *Journal of the American Statistical Association,* Sept., 1931, pp. 353–356, a recent book by

E. B. Kurtz: "Life Expectancy of Physical Property," which Dr. Glover says is the first book to his knowledge "published in English dealing to any extent with the subject of life tables for physical property."

There has been a great deal of activity in the past two or three decades in adapting statistical methods for the solution of *engineering problems.* This activity has been especially manifest in New York City. The work of Thornton C. Fry, W. A. Shewhart, Arne Fisher, E. C. Molina, and others, is known to a good many members of the Mathematical Association and the Mathematical Society. On a number of occasions, I have been asked to nominate a student adequately prepared in engineering and in statistical methods for a position in New York City. This combination is hard to find; an engineering student has about all his work prescribed, and a liberal arts student does not get enough engineering.

Recently T. C. Fry published the textbook "Probability and Its Engineering Uses." Dr. Fry is not wholly absorbed by the engineering aspect of statistics and probability, but has given a great deal of thought and attention to the theoretic problems and perplexities of probability. So the book is well worth reading as an all-around good book on probability. Two quotations from this book will give some conception of the type of problem attacked: p. 240, "Suppose that telephone calls, each of length T, occur individually and collectively at random, the calling rate being n per unit time. What is the probability that, at the instant t, there are exactly j in progress?" P. 229, "A retail chain store, with limited storage facilities, sells on the average 10 boxes of dog-biscuit per week. The usual practice is to stock up Monday morning. How many packages should be adopted as the standard Monday stock, in order not to lose more than one sale out of a hundred?"

Both T. C. Fry and W. A. Shewhart are on the technical staff of the Bell Telephone Laboratories of New York City. Dr. Shewhart has sent me about a dozen monographs printed as Technical Publications of the Bell Telephone System in the years 1926–1931. These treat such topics as

"Economic Quality Control of Manufactured Products." A product is systematically sampled, the results recorded and plotted. The variations are then studied and compared with normal variations as determined from the theory of chance. If the variations become excessive, the machinery is examined to locate a cause for defective output.

I pass now from strictly utilitarian applications of statistics to the application to science as a whole. In the last few decades a great unification of the various sciences has taken place. Scientists have gradually come to the realization that *many of the biggest problems in all sciences are fundamentally statistical in nature.* This followed the discovery that the atom is not an indivisible unit but a replica of the solar system with planets whirling about a central body. It is, indeed, extremely doubtful whether there is any ultimate unit of matter or energy. In a paper: "Das Weltgebäude in Lichte der Ätherspannung," *Annales Academiae Scientiarum Fennicae,* Series A, Vol. 30, HJ. Mellin, associate editor of *Acta Mathematica,* conceives of an infinite succession of ethers, whose particles have sizes corresponding—roughly speaking—to infinitesimals of the first, second, and higher orders. The so-called "*physical laws*" of any designated ether are then merely the "*statistical laws*"—due to mass action—in the ether of next higher order. At all events—whether we accept this theory or not—the older mechanistic or deterministic theories have been abandoned. *Instead of inexorable physical laws,* we have merely *statistical regularities*—that is, *regularities* which are *somewhat irregular.* True, these irregularities may be of negligible importance in most practical applications.

The different branches of science have been, to a large extent, *knit together* by the statistical method. We must acknowledge, of course, that a science will have individual problems. Some of these will be qualitative. Some will be quantitative in the usual sense with a solution essentially exact. For other problems, the *margin of uncertainty* in in the solutions will be so wide that there will exist a *solution* only in the *statistical sense.* This is true even in physics and astronomy.

A notable book exploring the possibilities of unity in science has been written by A. J. Lotka of Johns Hopkins University; the title is: "Elements of Physical Biology." It was published in 1925, but plans were laid for the book in 1902 when Dr. Lotka was a student at Leipzig. In this book the first three illustrations were reproduced from Dr. Lotka's paper: "Two Models in Statistical Mechanics," published in the *American Mathematical Monthly,* Vol. 31 (1924) pp. 121–126. With extensive reference to workers in many fields, Dr. Lotka attempts to coördinate the results in a general approach to science through the medium of mathematics. The approach is indeed statistical. But a glance through the book might not reveal much of the usual machinery of statistics. The most predominant mathematical feature is the *differential equation* or sets of these equations—involving usually only the first derivatives. This, after all, is quite material. In physical chemistry, the equations to express mass action arise most naturally as differential equations to represent rates of change. Dr. Lotka's purpose was to extent to *physical biology* some of the principles of *physical chemistry.*

Right here, it may be well to note that physics and chemistry have already been brought closely together. Physics has always been, confessedly, a mathematical subject. Chemistry, as it progressed, realized the need of more and more mathematics, especially in the line of *differential equations* and *statistical methods.* When I studied chemistry, a good many years ago, I learned that two parts of hydrogen combined with one part of oxygen to form water. We wrote the equation $H_2 + O = H_2O$. It was a "presto-change" reaction, and that was the end of it. Perhaps I did not carry my chemistry far enough at the time; but that was what I got out of it. What is interesting now is this: "How much of the hydrogen and of the oxygen are actually converted into water, and how much hydrogen keeps its identity as hydrogen, how much of the oxygen its identity as oxygen? But this is a *statistical problem, a problem of finding distributions.* Dr. Lotka, by this problem, illustrates his line of march from physical chemistry to physical

biology. He writes in substance: Given 4 gram molecules of hydrogen [8 gms.] and 2 gram molecules of oxygen [32 gms.] and 100 gram molecules of steam at atmospheric pressure and at 1800° centigrade: the law of *mass action* is

$$\frac{1}{v}\frac{dm_1}{dt} = k_1\frac{m_2^2 m_3}{v^3} - k_2\frac{m_1^2}{v^2},$$

where m_1 = mass of steam; m_2 = mass of hydrogen; m_3 = mass of oxygen, all in gram molecules, v = volume, and the k's depend upon temperature. As a more general problem, he considers a system of n independent variables with j parameters and writes:

$$\frac{dX_i}{dt} = F_i\,(X_1, X_2, \ldots X_n, P_1, P_2, \ldots P_j),$$
$$i = 1, 2, \ldots n.$$

Dr. Lotka discusses the solution of such a set of equations by means of *infinite series* and a *"characteristic equation."* Various cases of *stable* or *unstable equilibrium* are discussed. Carried over into the biological field, it gives a *mathematical framing* for *biological phenomena*, such as mentioned by Herbert Spencer. Dr. Lotka quotes from Herbert Spencer's "First Principles," Chapter 22, Sec. 173: "Every species of plant and animal is perpetually undergoing a rhythmical variation in number—now from the abundance of food and absence of enemies rising above the average, and then by a consequent scarcity of food and abundance of enemies being depressed below its average. Amid these oscillations produced by their conflict, lies the average number of the species at which its expansive tendency is in equilibrium with surrounding repressive tendencies."

On the mathematical side to describe such phenomena we have *periodic functions* of the time like cos bt to give the *oscillations*. Such functions appear in solutions of simple differential equations, as do also *damping factors* of the form e^{-at}, $a > 0$. The product $K\,e^{-at}\cos bt$ would give a damped oscillation about a mean value—an oscillation vanishing for $t = \infty$.

Certain particular equations are considered in some detail. The differential equation

$$\frac{dX}{dt} = aX + bX^2$$

leads to

$$X = \frac{k}{1 + e^{-ct}},$$

which has been successfully fitted to the growth of population in the United States—also to a population of fruit flies. This curve has about the same general form as the curve

$$y = \frac{\pi}{2} + \text{arc tan } x.$$

It shows comparatively slow growth at the start, and also for large values of the time t. The population approaches asymptotically a maximum size. Dr. Lotka mentions in particular the work of Raymond Pearl in this connection. He also takes up Martini's Equations for Immunizing Diseases, the Ross Malaria Equations, and the work of W. R. Thompson on Parasitology reported in *Comptes Rendus* (1922) ; and gives in another field the passage of radioactive substances through their various forms with their *mean lives* ranging from 2440 years down to less than 5 minutes. The "Elements of Physical Biology" stresses biological phenomena — as it should. But Dr. Lotka has a decidedly *philosophical* cast of mind, and this is shown throughout the book, notably at the beginning. Here various definitions of "life" are reviewed to show their inadequacy. There is, indeed, little to show that there is any actual demarcation between the animate and the inanimate. Nature seem to present a substantial continuity; and it would seem reasonable to seek a general view of the whole through some principle of evolution. And we should expect the same general mathematical expressions to be usable throughout the whole structure, for depicting the cosmos and its many parts.

I have already referred to Dr. Lotka's paper in the *American Mathematical Monthly*, "Two Models in Statistical Mechanics." Here is given an interesting treatment of *reversibility*, a topic of fundamental importance in *thermodynamics*. A difference between thermodynamics and the newer statistical mechanics is pointed out. By the Second Law of Thermodynamics, the entropy never decreases. Dr. Lotka states: "Statistical mechanics admits such small departures, such momentary very slight decreases in entropy, when the system is very near equilibrium. Experimentally also these small fluctuations have been demonstrated by Svedberg, Smoluchowski and others." The difference between thermodynamics and statistical mechanics may also be pointed out as follows: Thermodynamics states that a system *always* moves from a less probable to a more probable state; statistical mechanics merely states that it *usually* does so move. Statistical mechanics *removes determinism* completely from the picture. It has been jocosely asked whether if a kettle of water were set upon ice, the ice would ever cause the water to boil. The answer of statistical mechanics is that it would; but that you would have to wait through billions of billions of geologic epochs to see it happen.

I must now pass on to improvements in *statistical technique* in the last few decades. These improvements are numerous and of vast importance. But for the most part they are decidedly technical—too technical, I have felt, for me to dwell upon this afternoon. I will merely mention one direction in which these advances have taken place, and refer you to some literature. The older and classical theories of statistics presuppose a large, or at least fairly large, aggregate of objects for investigation. Many investigators now are specializing on *"small samples."* In many cases, in practice, only a small number of observations or measurements are available. It is to be presumed that they convey some information. But a general revision must be made of formulas for probable error; and, indeed, many changes are necessary. When samples of N objects are drawn from a large population normally distributed, it is

known that the means of the samples, divided by the actual standard deviation of the parent population, are normally distributed. But the actual standard deviation is, in general, not known. The new approach to the subject dates from a paper by a writer under the *nom de plume* of "Student" appearing in *Biometrika,* Vol. 6 (1908–1909) pp. 1–25. Here the distribution for the ratio of the mean to the *calculated* standard deviation was set forth. R. A. Fisher, whose work on small samples is of fundamental importance, obtained the distribution of the *ratio* of the *calculated standard deviations* of *two samples*—using for convenience the logarithm of this ratio. R. A. Fisher's *"Statistical Methods for Research Workers"* (Second edition 1928) is an exposition of the newer methods of testing for significance, written mainly for those without much mathematical equipment. Paul R. Rider in the *Annals of Mathematics,* Vol. 31 (1930) pp. 577–628 has an excellent and comprehensive paper: "A Survey of the Theory of Small Samples." In the *Proceedings of the American Statistical Association,* March, 1931, there appear a number of papers showing recent developments presented by mathematicians at the annual meeting in Cleveland, O., December 29–31, 1930. These mathematical papers—under the title of "Statistical Methodology"—were presented by Dinsmore Alter, B. H. Camp, H. C. Carver, A. R. Crathorne, E. L. Dodd, G. C. Evans, Arne Fisher, Ragnar Frisch, Harold Hotelling, E. C. Molina, H. L. Rietz, W. A. Shewhart, and E. B. Wilson. A great storehouse for information regarding statistics, both theoretical and applied, is the journal *Biometrika,* edited by Karl Pearson.

I have told you a little about the modern uses of statistics in engineering and the so-called "useful arts." Next, I discussed statistics in its relation to sciences in general, as supplying a unifying factor. The statistical method has its use in astronomy, physics, chemistry, and biology. Briefly, I mentioned the philosophic aspect—this might have been elaborated at considerable length. And I have spoken of improvements in technique. But my own great interest in statistics is somewhat outside all of these

phases. Primarily I am interested in *statistics as mathematics*—I may add as *pure mathematics*. The theory of statistics is about the same as the theory of probability. But what is probability? To this, many answers may be given. As developed historically, the conception of probability contains much that is essentially non-mathematical, much that can easily give rise to endless controversies. In 1904, there appeared in *Mathematische Zeitschrift*, Vol. 4, pp. 1–97 an article on probability by R. von Mises of fundamental importance, entitled "Fundamentalsätze der Wahrscheinlichkeitsrechnung." Carefully avoiding for sixty pages all the definitions and conceptions usually associated with probability, R. von Mises lays down in *pure function-theory* a set of theorems which he then restates in the usual language of probability. By this means he successfully *takes the "probability" out of probability.*

A notable paper on probability appeared in *Fundamenta Mathematicae*, Vol. 4 (1923) pp. 34–71, "Nouveaux fondements du calcul des probabilités" by A. Lomnicki. In this paper a probability is merely the ratio of the measure of a sub-set to the measure of the set, measure being taken in the sense of Lebesgue or Carathéodory. On the theoretic side it seems to be a question whether or not it is expedient to regard probability as a *separate branch* of mathematics. When so regarded, it is customary to explain what is meant by "equally likely events" or to incorporate the conception in some axiom, or to accept this conception as a primitive and undefined element of the structure. On the other hand, if probability is not a separate branch of mathematics, we must be able to *restate in the language commonly current among mathematicians all the important propositions associated with probability.* I am inclined to regard the latter course as preferable. But a good deal remains to be done if this course is adopted.

SOME OBSERVATIONS BEARING ON PUBLIC SCHOOL MATHEMATICS

BY B. H. MILLER
Superintendent, Eagle Pass, Texas

1. *Primary Grades*

In this discussion the primary grades include grades 1–4 in the Texas public schools. The middle grades, 5–7, are referred to as the grammar grades, and the upper grades, 8–11, as high school grades. This grouping of grades is probably obsolete as to school organization in the larger towns and cities at this time; but for the purpose of this paper the segments as above outlined will do.

Observation of scores made on tests in thirty-nine different states last year convinces me that there is no generally accepted standard of achievement, grade by grade, to be expected in primary number work; or if there is such a generally accepted standard, then we are far from achieving anything like an approximation to that standard in the results being accomplished. Perhaps each local system is setting up its own standard of achievement; or worse still, perhaps each teacher is determining for herself what the achievement should be at each stage of the progress of the child through the school. It is hardly probable that we have arrived at the stage where a scientific standard of achievement has been set for each child at each step in his progress. Can we determine effectively in a scientific manner just what should be done at each stage through the school as a minimum essential? If so, can we establish that minimum standard as the basal test of our teaching in number work?

Observation of the instruction of hundreds of teachers in primary number work over a period of years convinces me that teachers are not now employing scientifically tested methods of instructing these primary children in number work; nor are they devoting a regularly agreed amount of time to number work in the grades taught. Should the

instruction be done in a manner scientifically determined and should a standard minimum amount of time be devoted to this instruction? Or should the time given to number instruction be determined by the needs of each individual child or group of children instructed?

Examination of the courses of study of more than a hundred colleges that issue credentials upon which certificates to teach are based has not indicated to me that colleges generally give much time to the problem of number work instruction in primary grades; and I believe that most of the work offered is done by professors of education whose major interest is not number instruction. Should departments of mathematics in colleges and universities make a scientific survey of the number work materials and methods in conjunction with the laboratory school of the department of education of the college or university?

Has instruction in number work in the primary grades been modernized and vitalized and correlated with the other activities of this segment of the work of the public schools? Or is it still just a messy lot of routine and drill that is being done in a more or less perfunctory manner because it is in the course of study and some time has to be given to it? And does the neglect of number work in the primary grades account for the dislike of mathematics in the upper segments of the school? Can it be said that failure to do a good job of number work in the primary grades is the prime cause of all the troubles that beset the teachers of mathematics in the middle and upper grades? Should a primary teacher not be gifted in primary work just as she is gifted in reading, music, penmanship, drawing, and any other item in the curriculum? If so, are we giving the required amount of training in this work? It seems to me that perhaps here is the key to much of our troubles in mathematics all the way through the school system. If this is in some degree correct, then what responsibility has the department of mathematics in the teacher training college or university for remedying the situation? Or should the full responsibility fall upon the school of education or the education department?

2. *Middle Grades*

From talks with hundreds of teachers of mathematics in the middle grades I have heard hundreds of times that one of the greatest difficulties they meet with is the fact that the pupils come to them unprepared or poorly prepared to do the work that is expected of them. Does this indicate that there is no understanding between the primary teachers and the grammar grade teachers as to what belongs in each segment? Or does it indicate that the primary teacher has failed to do what she has understood to be her assignment? Or is it just an alibi? Whatever it is, it does not contribute anything to the solution of the problem. We never solve problems by passing the buck.

Children in the middle grades are mentally prepared to solve certain types and certain forms of problems whether they have mastered all combinations and processes indicated for the primary grades or not. Of course, failure to master the combinations and processes will make for errors in the work and loss of time; but as these inaccuracies are overcome the progress should be improved. But teachers in the middle grades should know just what types and forms of problems are within the range of these children, and how to give instruction that will assure satisfactory achievement. Do they do it? Or is mathematics in the middle grades also treated as a necessary nuisance about which nothing much need be done? Here again, though not in the same degree, we find lack of uniformity as to what is to be taught and how much time is to be given to it. It is again a neglected field in too many cases. Why is this so? Is it that practically all the work falls in the field of arithmetic, a subject which it is thought that anyone can teach?

In the middle grades it is also true that our teacher training schools have perhaps given too little attention to a study of the materials and methods to be used in mathematics instruction. Here again perhaps the department of mathematics has not always coöperated with the education department in the problems involved. Perhaps those who have borne the brunt of deciding what is to be done have

been compelled to plan and propose without a clear and scientific leadership in the field of mathematics. Just arithmetic is perhaps too childish to appeal to the thinking of a trained mathematician. It is too simple for his highly trained mind; or else he is too far removed from the mental processes of the child. It may be that he could not come down to that plane of thinking. Still, the mastery of these simple processes is the foundation upon which all mathematical thinking is based.

Again, in the middle grades it is also quite possible that too little is being done to coördinate the mathematics with the other subject groups. Perhaps it would be better to say that correlation of arithmetic with the social studies has been neglected on the whole. For this reason, arithmetic is poorly taught to poorly prepared pupils by mathematically untrained teachers who see no way to modernize and vitalize this material and make it a reality to the child taught. Result, failure to master the processes of ordinary arithmetic. In other words, inability to implant into the mind of the child mathematical thinking of an elementary and useful nature.

Should the simple equation be taught in the elementary school? And should pupils be familiarized with use of letters as numbers? Should elementary mathematics be organized without regard to the old subject divisions of arithmetic, algebra, etc.? How much mathematics can be taught on the elementary level? Should mathematics be considered purely as a tool subject, or has it a distinct cultural value all its own that cannot be had elsewhere? It seems to me that these questions have not been answered in our present courses of study in mathematics in the middle grades. Who should answer them? Has the department of mathematics a responsibility here, or shall we again ask the professor of education?

Would it be an exaggeration to say that at present, broadly speaking, we have a fragmental and broken course in mathematics in the middle grades which rests upon an

uncertain foundation of neglected number work in the primary grades upon which to erect our secondary course in mathematics? Who can find the answer for me?

Finally, what should be the criteria for determining just what should be included and what excluded from elementary mathematics? How much of it should be tool material, or utilitarian only? How much should be cultural only or also cultural? Can these questions be answered in a scientific manner? If so, whose is the responsibility for finding the answers? This suffices to allow me to suggest that such information should be available to school administrators and supervisors. Such information ought to be in the hands also of every teacher of elementary mathematics.

3. *High School*

The present organization of mathematics in the high school, while indefensible, is sufficiently crystalized so that no time need be devoted to a discussion of the objectives there. It is not true, however, that the results obtained at this level are comparable to the uniformity of organization and courses offered. Results obtained range from very poor to excellent where the same identical courses are offered with the same texts in use throughout in the several schools studied. Why so great a difference in results obtained when aims, objectives, courses, textbooks, etc., are identical? Do students differ so widely from school to school?

Here again, it is probably true that there has been too little attention given to the training of teachers to teach mathematics. Even at times it is entirely possible that the teachers themselves have never been taught enough mathematics to know what to teach or how or when it can be taught. Perhaps more has been done in the training of high school teachers of mathematics than in the case of teachers of elementary mathematics. It is no doubt true that many colleges and universities offer excellent opportunities for pupils to learn mathematics well, also to learn well how to teach mathematics on the high school level.

That level at least approaches nearer to where a mathematician feels at home. But here the problem seems to be not how well we can and do train teachers to teach mathematics, but how many we train. Judging from the number of graduates of Texas colleges who have majored or minored in mathematics for their first degree, one would feel that perhaps the entire output would be required to man the mathematics departments of colleges. The result is that many people who are not trained to teach high school mathematics have to teach it. Why are there so few majors in mathematics in our colleges? Can the department of mathematics escape some considerable responsibility? Is it that various departments of our colleges compete for major students, and that the mathematics department always loses the contest for numbers? Is it that mathematics is more difficult or less interesting? Is it that college mathematics fails to appeal to the modern youth? Could it be that most of them have been soured on mathematics because of botched jobs done in the lower schools by poor teachers? Perhaps it is true that mathematics is one of the most poorly taught subjects in our public schools. If so we would not expect boys and girls just out of the public schools to be very much interested in college mathematics. What responsibility has the college mathematics department in rehabilitating the subject and thus opening the way for better achievement in mathematics from the kindergarten through the college and university?

THE TEACHING OF GEOMETRY IN A TEACHER-TRAINING CLASS

BY EDNA GRAHAM
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The teacher-training course in secondary mathematics in West Texas State Teachers College has as its prerequisite junior standing in mathematics. That is, the applicant for the course must have done creditable work in college algebra, trigonometry, analytic geometry, and the first year of the calculus. It is my purpose to discuss the parts of this course pertaining to the teaching of plane geometry from the standpoints of: (1) activities of the teacher and the pupils, (2) methods and plans, (3) current topics in geometry, (4) types of geometries, and (5) the place of geometry in the curriculum.

Ernst R. Breslich lists the activities under eight heads, namely: teaching technique, how to start the class period, selecting subject matter, taking an inventory of what the class knows, giving the pupil a broad view of the unit, studying the unit, testing, and recitation. We make a careful study of this chapter of Breslich's and place much emphasis upon magazine articles pertaining to these subjects. In order that the students may get a quantity of material without excessive reading, articles are assigned to individuals and reports are made to the class. Members of the class are required to take notes upon these reports.

To illustrate one activity of the teacher, I shall take "The Preview of a Unit of Area." Begin with the area of a square and take in order the rectangle, the triangle, the parallelogram, the polygon, and the circle. Little of this material is entirely new, for the student has met most of these areas in his arithmetic; therefore, the presentation in the preview will be largely intuitive. The student sees area as a whole unit; then the demonstration of the theorems on area are to be taken up in their logical order. This preview develops good listeners, gives a motive for the study of each separate theorem, and shows the student where he is going. I give the preview to a teacher-training

class in much the same way that I would expect them to give it to their high school students. Before the close of the course, I have them choose a unit and write out a preview of it for me.

Another important activity for both teacher and pupil is the assignment. Breslich says that the assignment should be preceded by the following steps:

1. Let the students read the discussion in the text or have the teacher and pupils develop it together.
2. A clear exposition is now given by the teacher; the right form and best arrangement are used; application is made.
3. Pupils are sent to the board to see if they understand.
4. A supervised study assignment is made; this gives the teacher another opportunity to assist those needing further help. In this work we make reference to methods of recitation in England and Germany as given by J. W. A. Young and T. P. Nunn.

We study the various methods of presenting plane geometry, namely lecture, analytic, synthetic, drill, genetic, question-answer, heuristic, text book, project, laboratory, supervised study, blackboard, Socratic, and contract. After the methods are understood, we visit at least six teachers of geometry; and the student reports according to the following outline taken in part from *A Course for Teaching Junior High School Mathematics*, by W. L. Shaaf: (1) lesson aims, (2) lesson plans, (3) organization of material, (4) instructional equipment, (5) special devices, (6) drill lesson, (7) appreciation lesson, (8) questioning, (9) teaching pupils how to study, (10) development lesson, (11) supervised study, (12) assignment, problems, projects, (13) tests, (14) atmosphere, (15) routine, (16) ventilation, and (17) discipline. I make these visits with the students, and we follow them up with classroom reports and discussions.

After the methods have been mastered and the reports have been made upon observations of the experienced

teachers at work, each student is required to give a demonstration lesson to the class on a subject of his own choice. One young man chose to review triangles; he did it by means of flash cards. Members of the class became quite enthusiastic as they quoted the theorem or the exercise and gave imaginary proofs—I mean proofs without drawing figures. The student-teacher let blue represent the part given and red the part to be proved.

A young lady gave her lesson on the teaching of the word "locus." She made her illustrations out of cardboard for the following problems: (1) find the locus of a point two inches from a given line; (2) find the locus of the points of a pair of shears as they open and close; (3) find the locus of a point two inches from a circle whose radius is three inches; (4) find the locus of the vertex of a right triangle with a given hypotenuse.

We discuss the plans for projects, and a student is permitted to substitute a project for his term paper. Booklets, posters, and projects are required of every student in some schools in order that the school may be able to send the best of these to annual school exhibitions; however, there may be other reasons. Miss Housen of Enid, Okla., says: "These projects have been the means of creating greater interest in mathematics without in any way lowering the standard for the regular work." Following are some of the project subjects that have received some attention in our teacher-training course: geometry in textiles; in tiling, linoleums, and rugs; in home furnishings; in advertising; in art; in court; in nature; in astronomy; in bridge building; in windows; and in musical instruments. (A group of projects done by high school students was exhibited.)

We give some attention to that system of individual instruction that is often named the "Dalton Plan" and sometimes called "Contracts." My students make up a list of work sheets with the assignments for a unit of work: for example, the angle unit, the triangle unit, or the circle unit, etc. This idea is not original; it came to me from the Ardmore High School through an article in the March, 1930, number of *The Mathematics Teacher*. This article

was written by James H. Zant of Durant Teachers College. This is his plan: a work sheet gives the pupil his assignment for a three-week period. It was designed so that all pupils covered the "minimum essentials," and was intended to include enough to tax the capacity of the best. The starred problems required of everybody were selected as those with which the pupil would have to be familiar in order to go on with the course. The standards set for the marks A, B, C, and D were purely arbitrary. Mr. Zant distributed the sheets to the students and the work was begun during the class period. The puplis could get help from the teacher or any of their fellows. As soon as the pupil learned a proposition, he was at liberty to explain it to the teacher or to one of the monitors; the pupil's grade was marked in the proper column. Results, mistakes, and recommendations are given in Mr. Zant's paper, but I shall not quote them here. I have required my students to make their sheets in the form of contracts. The "C" contract includes the "minimum essentials" in the three-weeks work. The "B" contract contains all propositions, exercises, and corollaries in the "C" contract plus ten per cent more work. The "A" contract contains all of the "B" contract plus 10 per cent more work. The high school student is advised to sign the contract that is within his ability. Most of them want to sign the "A" contract at first; however, they are soon willing to accept the contract that they can do. The checking system is an important part of the contract and is usually worked out by the individual teacher. This work gives my students a good review of the subject matter of plane geometry and shows them how to organize and emphasizes the various phases of the subject. These students are all mathematics majors and know the subject matter, but they have little notion about the difficulties and values of the different parts of a unit.

We have also devoted a part of the time to the study of a one-year course in plane and solid geometry, and have read the articles of John C. Stone of Montclair, New Jersey, in one of which he says: "As to the logic value, I see no reason why a combined course might not be as

good or even better than a course in plane geometry alone. In order to do this, we can reduce the number of theorems of plane geometry to those that are clearly fundamental in building up a logical sequence. We could then give enough solid geometry during the latter third of the year to develop some sense of the limits of plane, an appreciation of some phases of geometry carried to three dimensional space, and some power to interpret three dimensional figures represented on a plane." In *The Mathematics Teacher,* March, 1931, Charles A. Stone of the Laboratory Schools of the University of Chicago says: "Probably no question is receiving more attention today than that of the reorganization of plane and solid geometry." He names three ways that we can preserve solid geometry:

1. Teach it as a separate unit.
2. Fuse it with plane geometry.
3. Teach it intuitively.

Mr. Stone mailed two hundred fifty questionnaires to teachers of geometry and received one hundred forty replies. To the question: "Do you feel that solid geometry should be taught in the high school?" one hundred eight replied, "yes"; and to "Do you favor a one-year course in plane and solid geometry?" eighty-eight replied, "no." Too many failures in a one-year course was the principal reason given.

In plane geometry, as in all other fields of subject matter, "teachers are turning to objective tests as the only reliable means of determining the efficiency of their instruction." For this reason, we work with the aims and purposes of testing to improve instruction:

1. Geometrical ability tests are studied for the purpose of properly grouping pupils.
2. The inventory test informs the teacher concerning the background of the student.
3. The standardized achievement test enables the teacher to make comparisons.
4. The diagnostic test shows the need of reteaching.

5. The unit-achievement test insures the teacher an opportunity to identify specific difficulties individually.
6. Completion tests, multiple-response tests, matching tests, true-false tests, choice-of-answer tests, all are studied because less writing is required and more time is given for thinking.

My students are required to write out the types of tests as listed in Breslich's *The Teaching of Mathematics in Secondary Schools* and they are also required to make a set of each type.

It is impossible for me to discuss, in this brief outline, all that we do in the teacher-training course in plane geometry. We study the types of geometry, Euclidean and non-Euclidean, using Schultze's *The Teaching of Mathematics in Secondary Schools* and Bussey's article in *The Mathematics Teacher,* December, 1922. Both authors show how Euclid, Lobatschewsky, and Riemann differed in their notions of the fifth postulate of Euclid. This study gives the class a viewpoint of the fifth postulate that they have never had before.

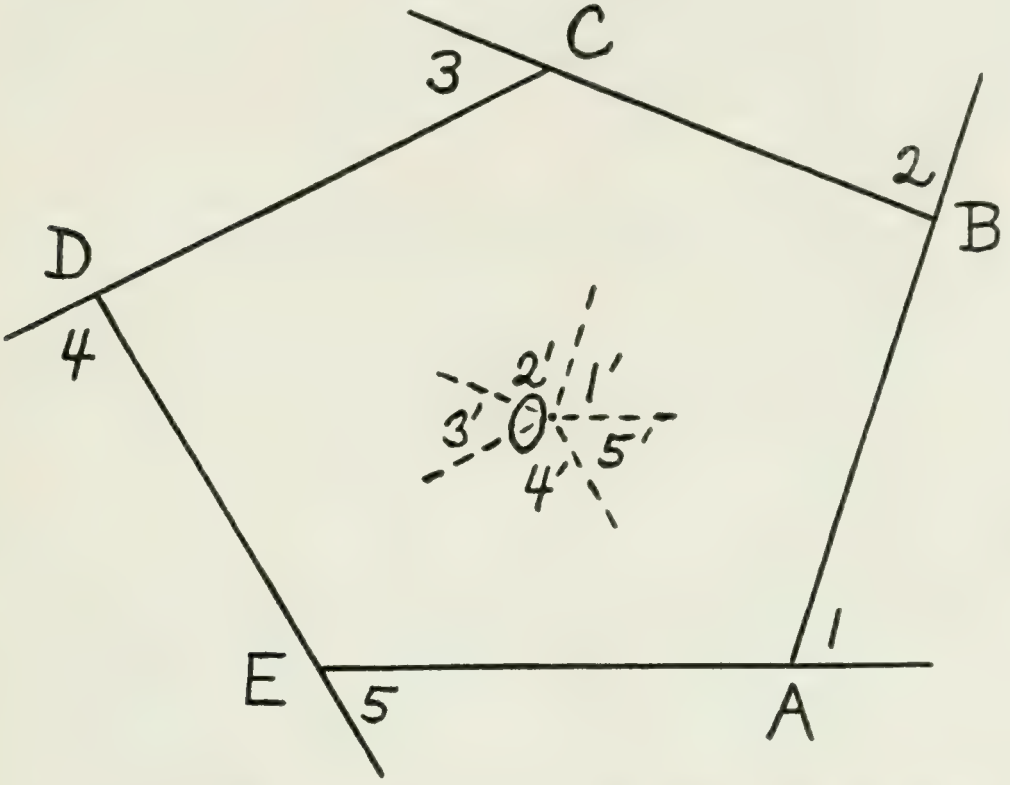
To be sure, I do not expect them to go out into the teaching field and give this information to their high school students. However, they can be better teachers of Euclidean geometry if they know something of the non-Euclidean geometry. Their major interests in the course are Euclidean geometry and the methods of presenting this to the class: intuitive, laboratory, etc.

We have a few lessons on the fourth dimension. Preparatory to this, we read *Flatland* and the articles in the *Scientific American,* 1920, together with three articles from *The Mathematics Teacher:* "Teaching Fourth Dimension and Hyperspace," March, 1926; "Reflections on Fourth Dimensions," December, 1925; "The Fourth Dimension," January, 1931.

About three lessons are given to a translation of Carlo Bourlet's *Motion Geometry.* I show the class that the emphasis is placed upon visualizing the ray, symmetry,

translation, rotation, and homothetic similitude. To illustrate, I shall give one exercise and one theorem:

The sum of the exterior angles of a polygon, obtained in the prolongation of the sides in succession, is equal to four right angles.



Along the guide AO, translate the angle 1 to the position of angle 1′. Similarly, translate angle 2 to angle 2′. Do the same with angles 3, 4, and 5. 4 right angles = angle 1′ + angle 2′ + angle 3′ + angle 4′ + angle 5′ = angle 1 + angle 2 + angle 3 + angle 4 + angle 5.

When two angles have their sides parallel, there are three possible cases: (1) OA and $O'A'$, OB and $O'B'$ are parallel and similarly directed. The angles are equal and coincide by translation.

(2) OA and $O'A'$, OB and $O'B'$ are parallel and oppositely directed. The angles are equal by the translation which brings O to O'. AOB coincides with the vertical angle $A''O'B''$, or the angles are symmetric with reference to I, the mid-point of OO', and they coincide by rotation of 180° about I.

(3) OA and $O'A'$ are parallel and opposite in direction; OB and $O'B'$ are parallel and similarly directed. The angles are supplementary; by the translation which brings O to O', AOB coincides with $A''O'B'$, the supplement of $A'O'B'$.

For the past three years our demonstration school has been experimenting with laboratory geometry and my students have been observing the work. They have offered the following criticisms of the course:

Favorable:

1. Laboratory geometry involves individual work, experimental work, and construction.
2. It supplies that which is essential in sense perception.
3. It teaches students how to read, how to discriminate between that which is assumed and that which is to be done.
4. Students observe laws of geometry before they do logical thinking.
5. Activity is directed with objective material.
6. They discover by experimenting.
7. There is a satisfaction in their accomplishment.
8. It encourages pupils to think for themselves, to judge results, to concentrate, and to ask for assistance only when they cannot do the thing for themselves.
9. The method is different, but not the content.
10. The student is led into a spirit of investigation and discovery.

Unfavorable:

1. It is the lazy man's way of doing geometry.
2. Sometimes it develops into a mechanical drawing course.
3. There are too many calls: "Come and help me with this construction." Too much assistance is given.
4. It makes dependent rather than independent students.
5. Not enough plates are made from the exercises. The work is largely reading and drawing according to directions.
6. It deals too much with drawing and not enough with logical geometry.

Our teacher-training course in secondary mathematics is a three-hour course and covers the principles and problems of all high school mathematics. We use Breslich's *Technique* and *Problems* as our basic texts, supplemented by the reports of the various committees on secondary mathematics. Not all the time is devoted to geometry, as one might infer from this paper. The last problem in the course is the making of a course of study in compliance with recent recommendations and modern trends in education. Thus an earnest effort is made to present to the teacher-in-training the current topics and problems in secondary mathematics in general and geometry in particular, in such a way that his own teaching of the subject will not be formal and intellectually stagnant; but that there will be a definite purpose in his teaching, that it will be logical, precise, and full of thought.

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THE UNIVERSITY OF TEXAS

FRESHMAN TESTS ON HIGH SCHOOL ALGEBRA AND GEOMETRY

BY MARY E. DECHERD

For the last three years tests on High School Algebra and Geometry have been given to the freshman classes in Mathematics 301 and Mathematics 302. The papers have been graded by the various instructors, and the results have been compiled and sent to the high schools concerned.

In October, 1931, these tests have again been given, with the following results. For the first time the records were made separately for Mathematics 301 and Mathematics 302; that is, for those who are taking Trigonometry and for those who, having had Trigonometry in high school, are studying Analytics in the University. Also, figures will be given showing separately the records of those students who took Algebra during the last year in high school and of those who did not. The totals are first given for the combined freshman classes and then for Mathematics 301 and Mathematics 302 separately. For the sake of comparison, last year's figures are given also.

	1931	1930
Number of schools	369	361
Number of students	972	910
Number of students passing	350	261
Number of students failing	622	649
Per cent of students passing	36%	28.7%
Number of students reviewing algebra in high school	209	
Number of students not reviewing algebra in high school	763	
Per cent of students passing who reviewed algebra in high school	64.6%	
Per cent of students passing who did not review algebra in high school	28%	

MATHEMATICS 301

Number of students		706
Number passing who reviewed algebra in high school	95	
Number passing who did not review algebra in high school	136	231
Number failing who reviewed algebra in high school	56	
Number failing who did not review algebra in high school	419	475
Per cent of students passing		32.7%
Per cent of students passing who reviewed algebra		62.9%
Per cent of students passing who did not review algebra		24.5%

MATHEMATICS 302

Number of students		266
Number passing who reviewed algebra in high school	40	
Number passing who did not review algebra in high school	79	119
Number failing who reviewed algebra in high school	18	
Number failing who did not review algebra in high school	129	147
Per cent passing		44.7%
Per cent passing who reviewed algebra in high school		69%
Per cent passing who did not review algebra in high school		38%

The number who made 90 or above on the tests has increased from 40 in the fall of 1930 to 64 for this year. Of the 64, 43 are in Mathematics 301, and 21 in Mathematics 302. That is, 6 per cent in Mathematics 301 and 7.9 per cent in Mathematics 302 made 90 or above. 6.6 per cent of the entire freshman class made 90 per cent or more on the examination.

The list of students follows, by schools:

MATHEMATICS 301

Alvin High:
Peebles, Howard 95
Austin High:
Benson, L. Mac 100
Dougherty, Burkes P. 99
Ford, Walter 95
McLaurin, Farrior 100
May, Elouise 99
Storm, Joe 95
Wunderlich, Wilma 97
Bangs High:
Puliam, Jack 93
Brooklyn (N.Y.):
James Madison High
Azelrod, Maitland M. 95
Bryan:
Stephen F. Austin High:
Walker, Shirlireid 100
Chicago, Ill.:
Senn High
Baldwin, R. B. 100
Claude High:
Davis, Hugh B. 93
Dallas:
Forest Ave. High
Cone, C. S. 90
North Dallas High:
Griffin, Franklin 96
Pope, Alex, Jr. 92
Oak Cliff High:
Robinson, J. Cliff 90
Sunset High:
French, Sims 95
Hall, Hugh Rather 93
Morrison, Walter 95
Woodrow Wilson High:
Smith, Ted L. 95
Platt, Earl 100
Fewell, Emmajane 95
Hockaday School:
Levy, Jean 97
Del Rio High:
Carl, Emory 92
Durango (Col.):
Pomeroy, Jennie C. 90
Eagle Pass High:
Stevenson, Ruby 90
Galveston:
Ball High
Bevil, Newton 90
Robinson, Evelyn 95
Granbury High:
Hines, James 95
Houston:
San Jacinto High
Crow, Fannie 90
Little Rock (Ark.):
Strong, Elizabeth 90
Lima (Ohio):
Henderson, W. G. 97
Marshall High:
Schulze, M. C. 90
Mercedes High:
Vann, Estelle 93
Merkel High:
Richardson, Elvis 90
Miami High (Florida):
Manly, Bob 90
Post Oak Grove:
Traweek, J. C., Jr. 93
Port Arthur High:
Mayer, J. S. 97
San Antonio:
Brackenridge High
Stewart, John H. 90
Shreveport (La.):
Byrd High
Richardson, John A. 95
Waco High:
Eleon, Ashton 90
Pratt, Derrill, Jr. 100

MATHEMATICS 302

Austin High:		Noel, Ernest	90
Boothe, Nellie Marie	95	Houston:	
Haston, Archie	92	St. Agnes Academy	
Otting, Dorothy	100	Angenend, Mary Agnes	95
Ravey, Hubert	92	Lawrenceville (N.J.):	
Whittaker, Lowry	96	Cheatham, Donald P.	95
Bucks Co. (Pa.):		Marshall:	
George School		Warren, Grace	90
Sayford, Mary Helen	100	Matador:	
Chicago:		Williams, Chas. Sumner	90
Senn High		Middletown (N.Y.):	
Harper, Charles	94	McNeely, I. R.	91
Dallas:		San Antonio:	
North Dallas High		Brackenridge High	
Martin, Mark	100	Klass, Henry	95
W. Wilson:		Main Ave. High	
Smith, J. B., Jr.	90	Pease, Gordon	92
Lincoln, J.	92	Sherman High:	
Fort Worth:		Jefferson, Margaret	97
Central High		Stamford High:	
Dalson, Max	97	Bunkley, Dorothy	96

In several other respects the grades are higher than last year. Note the following:

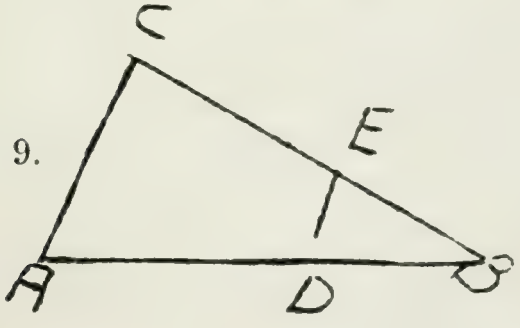
Grades Between	1930	1931	M-301	M-302
50– 59	138	153	106	47
60– 69	105	126	47	79
70– 79	62	89	63	26
80– 89	54	71	46	25
90–100	40	64	21	43

Also the per cent who made 50 or more is 51.7 per cent this year in comparison with 43.8 per cent last year. 82.7 per cent of Mathematics 302 students made 50 or more, while 40 per cent of Mathematics 301 students rated 50 or above.

Special attention should be called to the last two per cents in the first statistics given. It is noteworthy that a semester of Algebra during the last year of the high school should create such a difference in passing per cent. Of course there are, I presume, other contributory causes, but the figures are at least significant.

Of the larger cities, Galveston leads with 70 per cent passing. Dallas comes next, then Austin, and Fort Worth. Specially high averages are made by the North Dallas, Wilson, and Sunset schools in Dallas, while Fort Worth Central and San Jacinto, Houston, maintain their usual good standing. Of the small schools, note the good record of Lockhart, Eagle Lake, Denison, and Marshall. Many of the schools with the best records have evidently stressed taking Algebra during the last year of high school residence.

Copies of two examinations follow:

1. Simplify $\left(\frac{2}{3}\right)^3$, $-\sqrt{25}$, 65x0.
2. Simplify $a - \{b + c - d\}$, $\frac{1}{5 + 2/7}$
3. Factor $16 - y^2$, $a^2 - 3ab + 2b^2 + a - 2b$.
4. Simplify $(\sqrt{7})^2$, 3 x 2, $\frac{1}{\sqrt{2}}$
5. Solve for x: $2x + 3 = 7 - 1x$, $\frac{2}{x} = 5 + \frac{4}{x}$
6. Simplify $\frac{x^2y^7}{x^4y^3}$, $\sqrt{x^2}\sqrt{x^4}$
7. Solve $2x^2 - x - 3 = 0$
8. Solve simultaneously: $3x - 4y = 4$, $2x + 2y = 3$
9. 

In the triangle ABC, DE is parallel to AC, and $AD = 10$, $DB = 4$, $BE = 3$. Find BC.
10. Find the value of $\frac{2}{3} + \frac{7}{11} - \frac{3}{5}$

1. Add $2\frac{1}{3} + 5.6$. Get result as common fraction and decimal.

2. Divide $\frac{3\frac{1}{2}}{4\frac{1}{5}}$

3. Simplify $\frac{(9x^2 - 4y^2)}{3x + 4y}\ (3x - 4y)$

4. Find $\sqrt{8}$ to two places

5. Solve the equation $2\frac{1}{2}\,x = 5$

6. What is $\frac{a}{3 + b^2}$ equal to if $a = 4$, $b = 3$?

7. Find the value of $\frac{2}{\sqrt{2}}$

8. Find the altitude of the rt. $\triangle$ dropped from the right angle when sides are 3, 4, 5.

9. Solve $x^2 - 6x - 16 = 0$

MATHEMATICAL PARAGRAPHS

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The majority of high school pupils do not study. Much time is wasted in discussing the why and wherefore of teaching them how to study because they do not study at all. Any topic or skill, to become meaningful to a pupil, should recur many times in increasing difficulty. Absence of preparation on the part of pupils and class periods shortened either by untrained teachers or by extra-curricular activity interruptions, or both, prevent this necessary recurrence of topics and naturally they do not become meaningful; yet mathematicians wonder why the pupils confuse $\frac{x}{2}+\frac{x}{3}=5$ with $\frac{x}{2}+\frac{x}{3}+5$. Instead of fretting over the poor preparation of university freshmen, mathematicians should marvel over the fact that these freshmen, while not knowing an equation from an algebraic expression, do know that something about getting rid of fractions is involved in each of the above examples.

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Pupils who would not steal a dollar find it fashionable to steal on tests.

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Not long ago the writer published the following:

"Easy word problems are preferable to exercises; for instance, instead of $x/3=4$, 'One-third of the cost of a chair is \$4.00; find the cost of the chair' gives the equation more meaning. The type: 'A girl bought a dress and a hat for \$42.00; she paid $\frac{2}{5}$ as much for the hat as for the dress; find the cost of each,' is endless. Pupils can be taught to make problems for each other. They buy caps, gloves, wheels, radios, etc. Possibly they have a spending account or a budget. At least they will be more interested

in the prices of the things they buy than in the relationship between two abstract numbers."

Word problems do offer the most sensible approach to algebra. In fact, as far as a high school pupil's thinking goes, algebra should be the solving of word problems—all of the algebraic operations are merely means to this end—yet in much of the advanced mathematics and in a great part of the other sciences "playing with the symbols"—not always knowing the meaning of these symbols, perhaps not even knowing what the time-worn x stands for—is what usually counts. A master mind has set up the symbols. The graduate students, the young engineers, the blue-print workers, etc., for the most part, merely "play with the symbols."

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A one-year course in high school algebra, in some instances one year of general mathematics including all of the mathematics to be taught in high school, is being advocated. If those who try to teach mathematics are to know the subject only in a mechanical way, one year is too much. Otherwise, two or three class periods each week throughout the years of junior and senior high school are not too many to devote to clarifying the thinking which has been done outside of class. That some thinking, some studying, must be done by each individual who expects to approach an understanding of mathematics is proved by the fact that all true mathematicians have studied endless hours; and, also, the point is proved negatively by the countless number who, not being willing to work along mathematical lines, enjoy believing that they "can't learn mathematics." Of course some pupils can accomplish more in an hour than others can, and, while the mathematical ability of a pupil is not always measured by his speed in manipulating the different algebraic operations, there is a point in mathematics, as in all sciences, beyond which the majority of people cannot go. That point, however, is too often reached in the pupil's imagination before it is reached in his ability. Frequently, parents aid the children to

establish this mental hazard by boasting that *they* "couldn't learn mathematics either."

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In the mathematics classroom where the questioning prerogative is transferred from the teacher to the pupils, one usually finds the most fruitful results. Pupils have to do some thinking before they can ask questions worthy of class discussion.

* * * *

Long assignments, especially in the senior high schools, encourage pupils to do some individual studying, save class time, and serve as an outline or summary of the different units of work. There is something so satisfying in "working ahead" that oftentimes the interest of the majority can be caught and held. "I have all of the problems in next Wednesday's assignment except one and I have a whole week to keep trying it," a boy, who had been a rather listless listener, proudly announced. Then he could sit in the back of the room and "listen in" only when he wished to while the instructor tried to help some of the pupils attack this Wednesday's assignment. After class he stopped to say: "Now I see that quadratic equations really have *sense*. I like the way you presented that to Jane." Had he been forced to listen, or, after working ahead, had he not created some background of his own, the *sense* to him might still have been lacking. Another day a boy of similar type, with a twinkle in his eye just as if the joke were on the instructor, stopped to say: "When you were answering Tom's question, you unconsciously gave me a clue to the problem I couldn't get. Now I am going home and work it."

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Because the bright pupil can do something for himself, he is usually neglected for the pupil who has to be helped each step of the way. A teacher who knows his subject

can, with flexible assignments, ability groupings, thought-provoking questions, etc., keep the bright pupil more or less interested, but is only incidental attention fair to teacher or to child?

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Boards of Education who would not employ a dentist to test a child's eyes think it is right and proper for coaches, principals or superintendents who have time for one or two classes, teachers who have majored in anything save mathematics, etc., to teach arithmetic, algebra, and plane geometry. They are more careful about selecting the teachers of solid geometry and trigonometry when the official, who has time for one or two classes, does not remember how to solve the problems in the text.

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While poor teachers are ubiquitous, they, because mathematics is a building subject, can in this field, perhaps, do more than the average amount of harm. Invariably the high school officials give the advanced work to teachers whom they consider superior. Suppose the teacher of advanced algebra is superior, what can he do for pupils who have been poorly taught in first-year algebra? The officials, if they would reverse the assignments, might obtain two surprising results: first, the youngsters of junior high school age might jolt some of the demoted teachers out of their time-worn methods of presentation; second, arithmetic and first year algebra teachers, if given advanced mathematics, would, perhaps, take their first courses in college mathematics, do some studying at home, or, to avoid being exposed by the merciless seniors, transfer to a department where they may be more capable—or leave the teaching profession.

* * * *

Teachers should insist that their pupils consider the reasonableness of answers, and approximate and check results. A pupil who blithely announces that the house in

problem No. 8 costs $290.00 and then, after looking in the answer book, just as blithely says that he *only* has the decimal point in the wrong place, knows as little of approximating results as he knows of decimal points. The same pupil will be satisfied with 13.2 for the square root of 1.7424, $(x - y)$ $(x^2 + 2xy + y^2)$ for the factors of $x^3 - y^3$, and 36 inches for the area of a rectangle. Answer books, like other non-essentials, may be used in moderation.

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A man sent his son to an excellent college. The son failed. The man in discussing the failure remarked that the son's high school diploma meant nothing. He spoke of salt, sugar, and coffee being in certified packages, each package weighing so much, having so much strength, etc. He felt that a high school diploma should "certify" that the graduate could do college work, and that the high school should be held responsible when he failed. He was considering neither the human element—a child cannot be weighed as can a box of salt; a boy's I. Q. is a variable—nor his own responsibility—he had not made his son study. But the interesting question is: Why do high schools scheme to fail a low percentage when their "sins will find them out"? If the teacher could be *trained* (know his subject and how to present it), *trusted* (not to abuse privileges), and *free* (to fail all who deserve to fail), half of the pupils would fail, the parents would be awakened, and school money would, eventually, be saved.